Pressure Fluctuations and Particle Dispersion in Liquid Fluidized Beds

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The performance of a liquid-solid fluidized bed is substantially influenced by various mutually interacting factors, such as flow regime, motion of fluidized particles, and voidage distribution; this gives rise to highly complicated, nonlinear and/or stochastic behavior of the bed. While some attempts had been made to stochastically analyze and model such behavior, they were based mostly on the Markovian assumptions and notion of classical Brownian motion (Yutani et al., 1983; Yutani and Fan, 1985; Fan et al., 1995).

Recent works on multiphase flow systems demonstrated that some of the phenomena taking place in liquid-solid fluidized beds are amenable to the analysis couched in the parlance of the fractional Brownian motion (fBm) characterized by the Hurst exponent H or equivalently the local fractal dimension $d_{F\ell}$ (Feder, 1988; Fan et al., 1990b, 1993). The model based on this concept was proposed originally by Mandelbrot and van Ness (1968) to identify the long-term correlation in a self-affine time series. It was postulated that the scaled range (R/S) analysis can be a means of estimating H for a given time series (Mandelbrot and Wallis, 1969a,b).

Obviously, the motion of fluidized particles in a liquid-solid fluidized bed is primarily driven by the fluidizing liquid through which mechanical energy is supplied to the bed by pumping. This motion, in turn, causes the particles to collide among themselves and with the walls confining the bed; it also mechanically agitates the fluidizing liquid, thereby intensifying the turbulence. Thus, it is highly plausible that the behavior of the fluidized particles and that of the fluidizing liquid in the bed are inordinately complex and may be nonlinear. Such complex and nonlinear behavior manifests itself in fluctuations of some of the state variables or properties of the bed such as concentration, temperature, and pressure.

In earlier works (Fan et al., 1990a,b), pressure fluctuations were measured only under steady-state conditions. The present work extends these earlier works to unsteady-state conditions induced by a step change in the flow rate. The resultant variations of the dispersion and distribution of the fluidized particles are correlated to the temporal variation of H estimated for particles of various sizes and compositions fluidized at different flow rates. The approach and results of the present work should be useful for identifying and diagnosing both an upset in the operating conditions of a liquid-solid fluidized bed or deterioration in the performance of such a bed functioning as a contactor or reactor.

Theoretical Studies

A liquid-solid fluidized bed expands when the fluidizing velocity undergoes a stepwise increase (Yutani et al., 1982). Note that heterogeneous expansion is followed by homogeneous expansion, termed relaxation (Slis et al., 1959; Fan et al., 1963).

Since the movement of fluidized particles appears to be irregular and random during relaxation, it can be assumed to be essentially stochastic (Yutani et al., 1982). For convenience, a fixed segment in the axial direction of the bed is identified as the test section, thus giving rise to two states, the inside and the outside of this section. The number of particles in the section is considered to be described by the combination of two random variables; one obeys the pure-death process leading to the binomial distribution, and the other, the birth-death process, specifically the queuing process, yielding the Poisson distribution (Parzen, 1962; Chiang, 1980). During relaxation, the former represents the random exit of particles from the test section due to the bed expansion, and the latter, the entrance and exit of particles attributable to the randomly fluctuating motion of particles. The stochastic process characterized by these two random variables can be explored in terms of the mean number of fluidized particles in the test section and the fraction of mean

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number of particles that have exited from the section (Chiang, 1980; Yutani et al., 1982).

Since the motion of fluidized particles in the test section is regarded as random, it is convenient to characterize particle velocity in terms of its variance σ^2. This statistic gives rise to the particle diffusivity D_p. In other words, if the movement of a particle is characterized by the distance L_i traveled by the particle in the time interval, i.e., the mean residence time of a particle in the test section $1/\lambda$ the relationship between D_p and $\sigma^2(L_i)$ can be expressed as (Yutani et al., 1982)

$$D_p = \frac{\lambda}{2} \sum_{i=1}^{k} L_i^2 f(L_i) = \frac{\lambda}{2} \sigma^2(L_i) \tag{1}$$

where $f(L_i)$ is the probability density function of particle velocity. Furthermore, based on the assumption that the "ergodic hypothesis" is valid for the system under consideration, $\sigma^2(L_i)$ can be described by the variance of particle number; at the new steady state, it is $\sigma^2(\infty)$ (Yutani et al., 1982). Naturally, the pure-death process vanishes at the new steady state, and the motion of particles can be expressed solely by the Poisson random variable for which the mean and variance are identical (Chiang, 1980). In other words,

$$\sigma^2(\infty) = \overline{n_\infty}, \tag{2}$$

thereby leading to

$$\sigma^2(L_i) = \frac{L^2}{\overline{\overline{n_R}}} \sigma^2(\infty) = L^2 \frac{\overline{n_\infty}}{\overline{\overline{n_R}}} \tag{3}$$

Thus, D_p is obtainable from Eqs. 1 and 3 in terms of the number of particles

$$D_p = \frac{\lambda}{2} L^2 \frac{\overline{n_\infty}}{\overline{\overline{n_R}}} \tag{4}$$

or in terms of the bed porosity ϵ_ℓ as

$$D_p = \frac{\lambda}{2} L^2 \left[\frac{(1-\epsilon_{\ell\infty})}{(1-\epsilon_{\ell R})} \right] \tag{5}$$

Note that the ϵ_ℓ can be determined by the pressure drop measured in the test section.

The pressure fluctuations obtained can be examined by means of the rescaled range, i.e., R/S analysis, thus yielding the Pox diagrams. Eventually Hs are recovered from the diagrams. The procedure for estimating H is illustrated in Figure 1; the details of R/S analysis are available elsewhere (Mandelbrot and Wallis, 1969a; Fan et al., 1991). The increments of fluctuations can be characterized through H as follows

$0 < H < 0.5$	; negative correlation (antipersistence)
$H = 0.5$	; no correlation
$0.5 < H < 1.0$	; positive correlation (persistence)

The increment of the time series is Gaussian and self-affine; its $d_{F\ell}$ is related to the H as (Feder, 1988)

$$d_{F\ell} = 2 - H, \qquad 0 < H < 1 \tag{6}$$

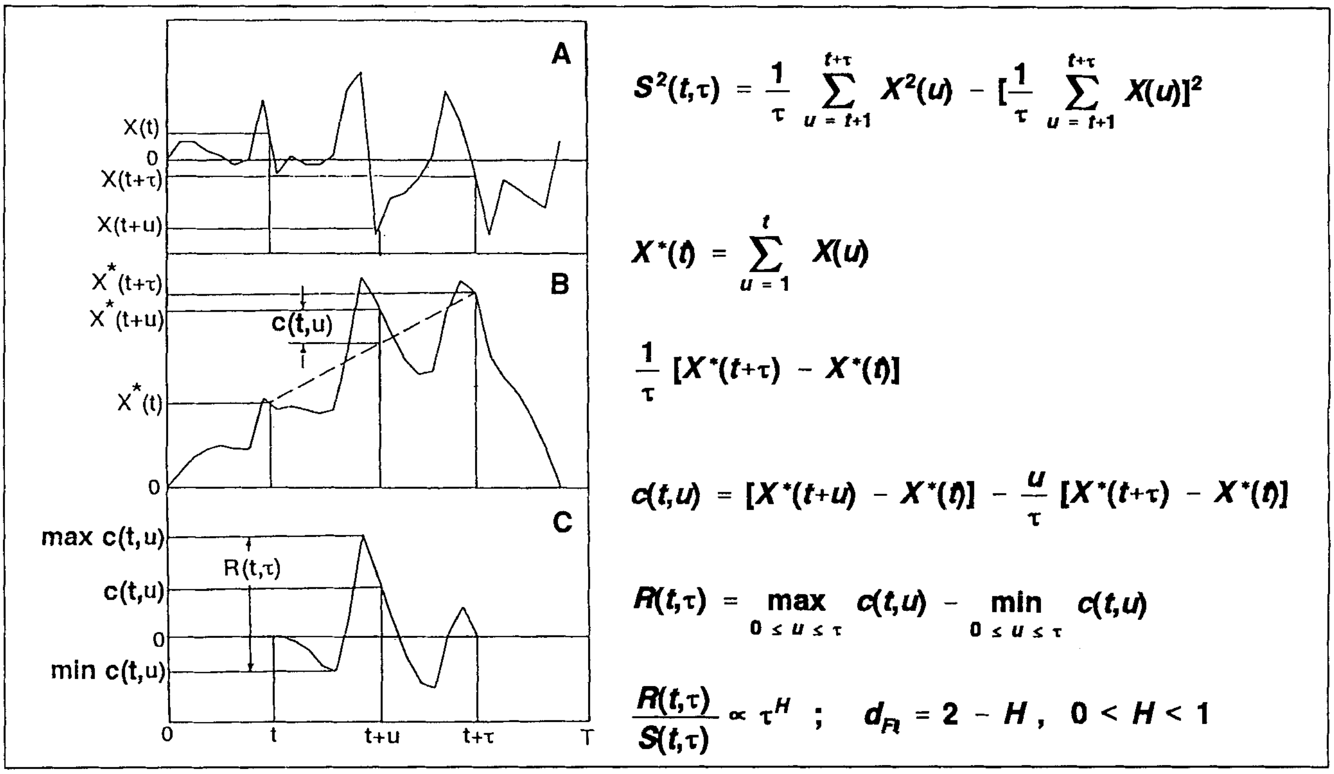

Figure 1. Construction of the sample range $R(t, \tau)$.

Experimental

Experiments were carried out in an acrylic column of 0.152 m in ID and 2.5 in height. The other details of the experimental apparatus can be found elsewhere (Kang et al., 1985; Kang and Kim, 1988). Water and spherical glass beads served as the fluidizing liquid and fluidized particles, respectively. The particle diameter d_p was in the range of 1.0×10^{-3} to 6.0×10^{-3} m, and its density ρ_p was 2,500 kg/m^3. The liquid flow rate U_ℓ in the range of 0.03–0.24 m/s was regulated by two valves and measured by a rotameter. The step change of valve opening generates the step change of U_ℓ.

The overall static pressure profile was measured by 15 liquid manometers located at 0.15 m intervals along the wall of the column and yielded the pressure drop profile; this gives rise to the bed height. Pressure fluctuations and their temporal variations were measured and recorded by a personal computer. The pressure fluctuations were transmitted from a pressure sensor; the semiconductor type was sufficiently sensitive to follow the dynamics in the bed. The sensor was located 0.2 m above the liquid distributor. If the particles are distributed homogeneously in the bed, the D_p can be estimated regardless of the location of the pressure sensor (Yutani et al., 1982).

The voltage-time signal from the transducer, corresponding to the pressure-time signal, was fed to the personal computer with a 32 bit A/D convertor at the selected sampling time of 0.03 s; a typical sample comprised 3,000 points. This combination of sampling time and sample points ensured that the full spectrum of hydrodynamic signals from the liquid-solid fluidized bed under either steady-state or unsteady-state conditions was captured.

Temporal variations of $\epsilon_\ell(t)$ or the particle holdup $[1 - \epsilon_\ell(t)]$ were determined from the relationship between $\epsilon_\ell(t)$ and pressure drop measured by a pressure sensor in the test section. The values of $\epsilon_\ell(t)$ or phase holdup measured under dynamic conditions have been known to be approximately the same as those measured at the corresponding steady state (Begovich and Watson, 1978; Kang et al., 1990, 1991).

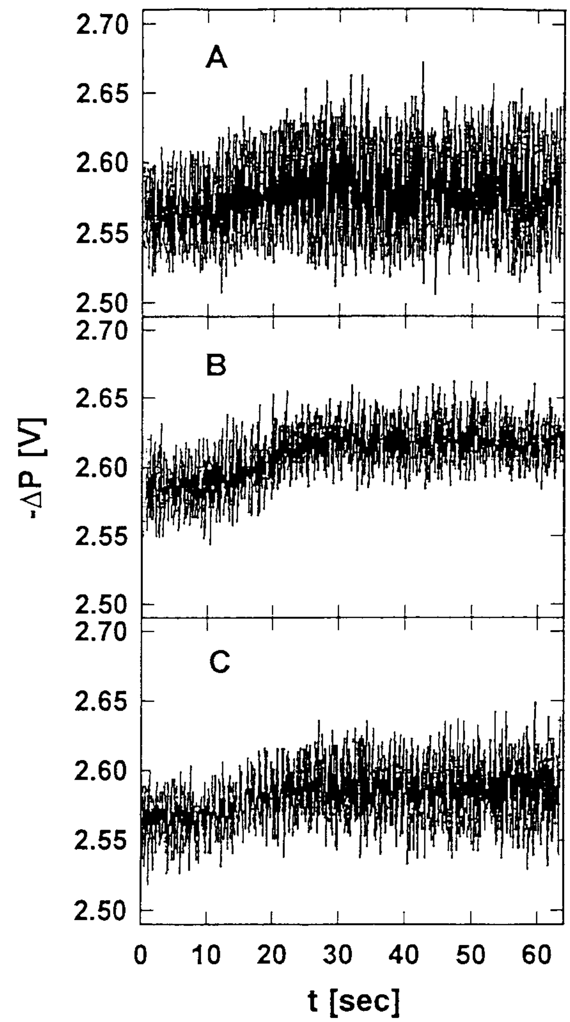

Figure 2. Unsteady-state pressure fluctuation signals for $U_{\ell,i} = 0.10$ m/s and $U_{\ell,f} = 0.16$ m/s.

	A	B	C
d_p[mm]:	1	1(50 wt. %)&3(50 wt. %)	1(50 wt. %)&6(50 wt. %)

Results and Discussion

Figure 2 exhibits typical pressure fluctuation signals during unsteady-state bed expansion from the initial steady state to the final steady-state conditions. The magnitude of the pressure fluctuations in the beds with two-size particles is smaller than that with uniform-size particles.

The variations of the pressure drop $(-\Delta P)$ and ϵ_ℓ were measured by the pressure sensor under either steady-state or unsteady-state conditions in the test section. For a given final liquid flow rate, the pressure-drop variations result in a unique relaxation point starting from different initial flow rates. The relaxation point can be determined, therefore, from these temporal variations in response to the change in the liquid flow rate (Yutani et al., 1982). The relaxation point in the bed with two-size particles can also be obtained according to the procedure employed in this work.

The reciprocal of the mean residence time of a particle in the test section λ has been evaluated from the temporal variation of $\epsilon_\ell(t)$ determined experimentally (Yutani et al., 1982; Kang et al., 1990); the λ exhibits a maximum with respect to the liquid flow rate U_ℓ in Figure 3. Kang et al. (1990) have reported that the λ attains its maximum at the intermediate liquid flow rate.

The D_p has been estimated from the experimentally determined values of λ and ϵ_ℓs (Yutani et al., 1982; Kang et al., 1990). The magnitude of D_p displays a maximum with respect to the variation in U_ℓ in all the cases studied (Figure 4). A comparison of Figures 3 and 4 indicates that the U_ℓ where the D_p attains its maximum coincides closely with that at which the λ is maximum (Kang et al., 1990).

It has been demonstrated that the stochastic or random component of pressure fluctuations in a liquid-solid fluidized bed under steady-state conditions can be characterized by H or $d_{F\ell}$ (Fan et al., 1990b, 1993). This characterization is adopted in the present study involving unsteady-state conditions. The value of H corresponds to the slope of the Pox diagram presented in Figure 5. As in steady-state conditions, the plots are essentially linear and that some of the rescaled ranges overlap or are compact for certain time lags. The overlapping and/or the break of the rescaled range is attributable to the presence of a periodic component in the

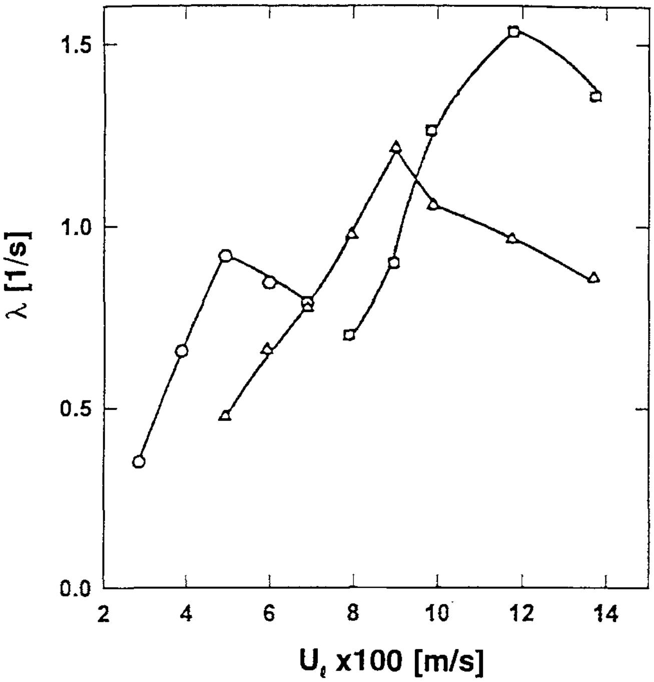

Figure 3. Effect of the liquid flow rate U_ℓ on the reciprocal of the mean residence time of particle in the test section, λ.

	○	△	□
d_p[mm]:	1	3	6.

pressure fluctuations (Mandelbrot and Wallis, 1969b). Note that the slope of the Pox diagram increases with time as the system approaches its final steady state.

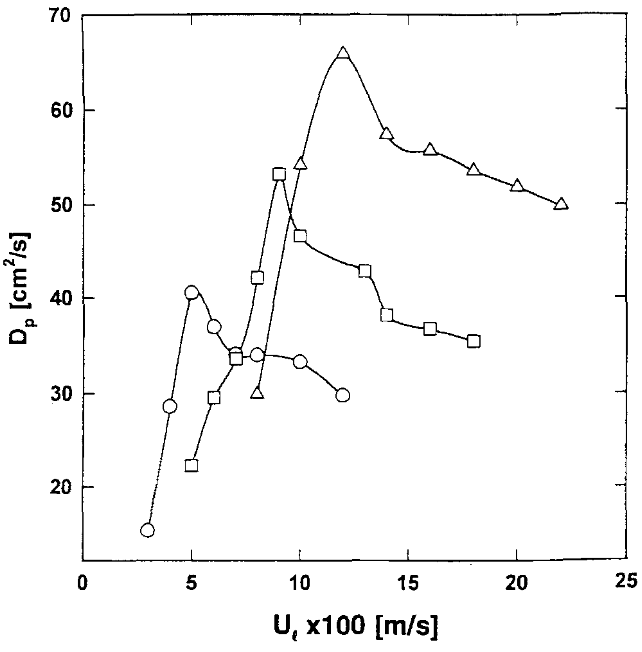

Figure 4. Effects of the liquid flow rate U_ℓ on the particle diffusivity D_p.

	○	□	△
d_p[mm]:	1	3	6.

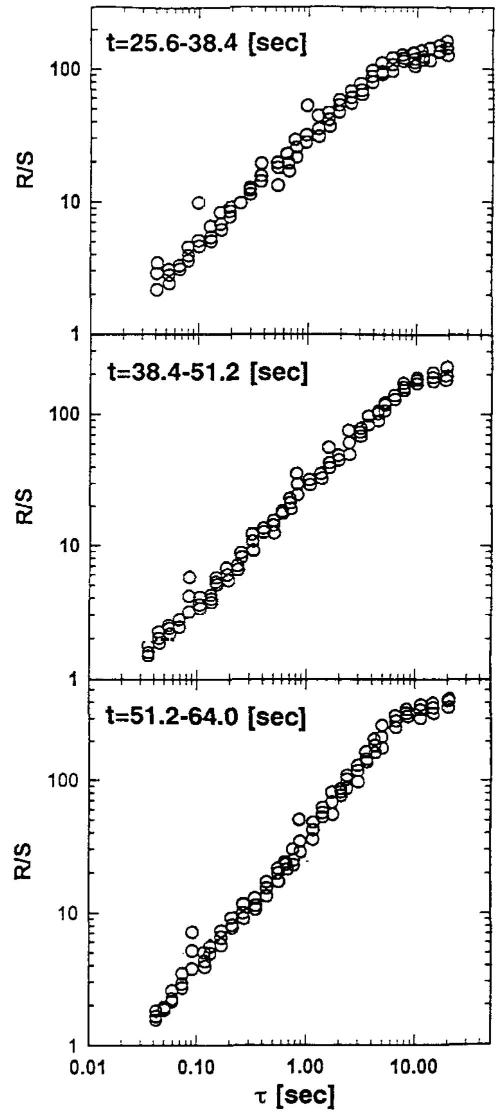

Figure 5. Pox diagram during unsteady-state expansion of the bed of glass beads, 0.001 m in diameter for $U_{\ell,i}$ = 0.06 m/s and $U_{\ell,f}$ = 0.14 m/s.

The relationship between H and ϵ_ℓ can be visualized in Figure 6. In each experiment, the temporal variation of ϵ_ℓ has been determined from measuring the pressure drop as a function of time after a step increase in U_ℓ. Note that the H increases steeply with an increase in the ϵ_ℓ during the period of heterogeneous expansion and increases gradually during the period of homogeneous expansion, i.e., relaxation; eventually, the H approaches an asymptotic value. As a consequence, the relaxation point or the onset of unsteady homogeneous expansion of fluidized particles can be estimated from this figure. In other words, if the Hs are calculated along with the bed expansion, a transition point from heterogeneous to homogeneous bed expansion can be detected.

Figure 7 demonstrates the effects of U_ℓ under steady-state conditions on H in the beds of uniform-size particles as well as in the beds of two-size particles containing an equal amount of small and large particles in terms of the mass; however, in terms of the number, the former is far larger than the latter.

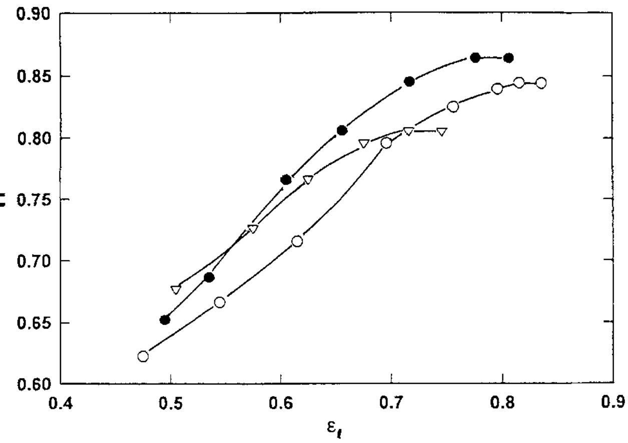

Figure 6. Effects of the temporal variation of the bed porosity ϵ_ℓ on the Hurst exponent H.

	○	●	▽
d_p[mm]:	1	3	6.

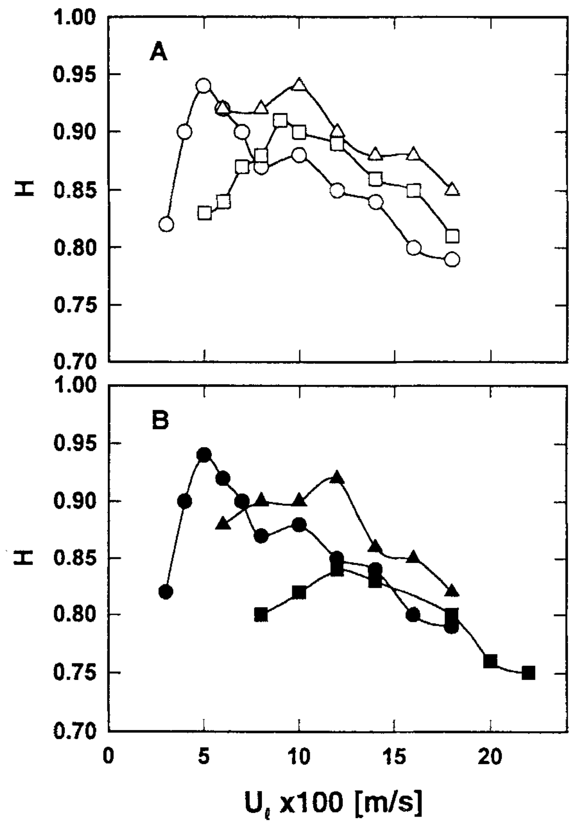

Figure 7. Effects of the liquid flow rate U_ℓ on the Hurst exponent H under steady-state conditions.

		A	
	○	□	△
d_p[mm]:	1	3	1(50 wt. %)&3(50 wt. %)
		B	
	●	■	▲
d_p[mm]:	1	6	1(50 wt. %)&6(50 wt. %).

The magnitude of H exhibits a maximum as the U_ℓ is varied in either bed. Comparing Figure 4 with Figure 7 indicates that the U_ℓ at which the H is maximum coincides with that at which the D_p attains its maximum. The larger the value of H, the more persistent the pressure fluctuations (Fan et al., 1993); thus, it is plausible that the pressure fluctuations are less irregular at the intermediate liquid flow rate. The motion of individual particles can be observed to be constrained by the neighboring particles at a reduced or low liquid flow rate: the particles tend to move in groups or in clusters. Nevertheless, the particles tend to move individually and essentially randomly when the U_ℓ is elevated or the particle concentration is reduced to an intermediate level. Under these intermediate conditions, the particles oscillate locally in relatively narrow ranges of frequency and amplitude (Kang and Kim, 1988; Fan et al., 1990a, 1993).

Figure 7 also shows that under steady-state conditions, the larger the size of particles, the greater the U_ℓ at which the H is maximum; moreover, the variation of H in a bed of two-size particles as a function of U_ℓ is similar to that in a bed of uniform-size particles. It can be seen that the Hs in the beds of two-size particles are in between those in the beds of small and large uniform-size particles when the U_ℓ is low. This appears to indicate that in the bed containing a mixture of different-size particles, the small and large particles independently contribute to a propensity of pressure fluctuations to persist because of relative slow movements in clusters. Nevertheless, the Hs are slightly larger than those in the beds of either small or large uniform-size particles when the U_ℓ is high (0.07 m/s $\leq U_\ell \leq 0.17$ m/s). It is expected that these particles of different sizes tend to interact nonlinearly and synergistically in contributing to such a propensity. It is worth noting that the H depends mainly on the mass rather than the number of particles.

Conclusions

The relaxation point or onset of relaxation signaling the transition from heterogeneous to homogeneous expansion of a liquid-solid fluidized bed can be detected from the temporal variation of the Hurst exponent H. The particle diffusivity D_p estimated from the temporal variation of the bed porosity ϵ_ℓ attains its maximum at the intermediate liquid flow rate where the H also attains its maximum. The H asymptotically reaches the maximum value as the bed approaches its final steady state.

Under steady-state conditions, the H exhibits a maximum with respect to the liquid flow rate U_ℓ in a bed of either uniform-size or two-size particles. When the U_ℓ is low, the small and large particles apparently contribute independently to the propensity of pressure fluctuations to persist as characterized by the magnitude of H; however, when the U_ℓ is high, these particles of different sizes apparently interact nonlinearly and synergistically.

Unsteady-state and steady-state conditions in the fluidized beds can possibly be distinguished through the estimation of H. Moreover, the H enables us to characterize fluidizing conditions by a single parameter; this would facilitate the development of an on-line control system of the fluidized bed.

Notation

$d_{F\ell}$ = local fractal dimension
d_p = particle diameter, m

D_p = particle diffusivity, m^2/s
H = Hurst exponent
L = location of the pressure sensor from the distributor, m
L_i = distance of the particle movement during time, $1/\lambda$
n = number of particles
$R(t,\tau)$ = sample sequential range for lag τ
$S^2(t,\tau)$ = variance of pressure fluctuations
U_ℓ = liquid flow rate, m/s
$X(t)$ = time series

Greek letters

ϵ = bed porosity
λ = reciprocal of the mean residence time of particle in the test section, 1/s
ρ_p = particle density, kg/m^3
σ^2 = variance

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Manuscript received Mar. 23, 1995, and revision received July 24, 1995.